REPORT CTSL– **14**

INJECTION FIELD CRITERIA FOR HIGH ENERGY SYNCHROTRONS

MATTHEW SANDS

JANUARY 5, 1961

SYNCHROTRON LABORATORY

CALIFORNIA INSTITUTE OF TECHNOLOGY

PASADENA

Report CTSL-14

CALIFORNIA INSTITUTE OF TECHNOLOGY

Synchrotron Laboratory

Pasadena, California

INJECTION FIELD CRITERIA

FOR HIGH ENERGY SYNCHROTRONS

Matthew Sands

January 5, 1961

Supported in part by the U.S. Atomic Energy Commission

Contract No. AT(11-1)-68

Contents

Abstract

Quantitative criteria are proposed for distinguishing "high" and "low" injection fields in high-energy accelerators. The distinction depends on the aperture scale as well as the synchrotron energy.

Acknowledgments

This work was aided by discussions with A. V. Tollestrup and R. L. Walker.

I Introduction

In a 300-Gev proton synchrotron, the cost of the injector may well be comparable to the cost of the main accelerator. The injection energy may have to be carefully chosen to obtain a proper balance between economic and reliability considerations. A low injection energy will reduce the cost of the injector, but will, in general, increase the cost of the main accelerator, by requiring larger aperture, more careful field corrections, etc., and may result in a lower reliability of operation. Too high an injection energy may increase the cost of the injector without a concommittant reduction in other costs, or with little gain in operational characteristics. In SL-10, a method was outlined for an optimization procedure applicable to a simple cascade machine (i.e., one in which the injector is a synchrotron similar, except for scale, to the main accelerator). This method is applicable in principle in the general case -- when the injector is a linac, a fast pulsing or an FFAG synchrotron -- but is less tractible analytically. It suffers in addition, the fault that it does not take into account the special problems associated with low injection energies. Fortunately, this refinement was not necessary for the design outlined in SL-10.

What is required, if other injection systems are to be considered, is some criteria which will give quantitative meaning to the concepts of "high" or "low" injection energies. This note derives a set of such criteria by a consideration of the magnetic field errors encountered at injection. The main conclusion is that a reasonable distinction can be made between "high" and "low" injection energies <u>and</u> that this distinction will depend on the size of the aperture of the main accelerator.

We consider separately the influence of (1) field errors at the design orbit, and (2) gradient errors.

II Field Errors

The vertical field (defined as the average value along the center of the aperture of a magnet) will vary from magnet to magnet due to variations in the remanent field, in the eddy-current fields, and in the magnet dimensions. Only the random variations about the mean value for the whole magnet are significant.

These random variations will produce a wandering of the equilibrium orbit such as that which is produced by random placement errors of the magnets. In fact, a variation δB in the vertical field of a magnet is equivalent to a placement error ε_{eq} given by

$$\varepsilon_{eq} = \ell_o \frac{\delta B}{B} \tag{1}$$

where $\ell_o = \rho/n$, and B is the nominal injection field.

The largest -- and least controllable -- contribution to the field errors is likely to be from the remanent field. These errors can be taken as independent of the total field at injection. For a given magnet, then, the equivalent magnet displacement varies inversely as the injection field.

The eddy-current fields will also be independent of the injection field strength, but will depend on the time derivative of the field, on the lamination thickness and resistivity of the iron, and on the properties of the vacuum chamber. We assume, for this discussion, that the variation in eddy-current fields can be made small compared with the variations of the remanent field.*

Dimensional errors δh in the height of the magnet pole gap will produce field errors δB given by

$$\delta B = B \frac{\delta h}{h} \tag{2}$$

where h is the gap height. The equivalent magnet displacement is then

$$\varepsilon_{eq} = \ell_o \frac{\delta h}{h} \tag{3}$$

which is independent of the injection field strength. If we consider only magnets scaled from the A.G.S. (or CERN) design, as suggested in SL-10, then h is proportional to ℓ_o and

$$\varepsilon_{eq} \simeq \delta h \tag{4}$$

---

* It is worth noting, though, that eddy-currents in the vacuum chamber depend strongly on the size of the aperture, favoring small apertures.

The constant of proportionality is about 3. It appears that this constructional tolerance can, in general, be set so that this error, too, is significantly less than the contribution from remanent field variations.**

For magnets whose whole cross section is scaled from the BNL or CERN designs, the remanent field will depend only on the variation of the coercive force of the iron, and the shuffling procedures, but not on the magnet scale. From the BNL and CERN data, one finds that the r.m.s. fluctuation in the remanent fields -- including those stray fields due to structures external to the magnet, which appear to be important -- are about 0.07 to 0.15 gauss. We assume that the materials control and handling procedures used on the 30 Gev machines are applicable to a 300 Gev magnet and adopt 0.15 gauss as a working figure. It is possible that lower values could be achieved by measurement and correction, but these possibilities also involve economic considerations and will not be taken into account here.

The assumption that the whole magnet cross-section is scaled from the BNL design does not permit the current density in the coils to be kept at the value used in the existing machines. If we assume a constant current density in the coils, the remanent field will vary somewhat with ℓ_o. In fact, one would expect that

$$\delta B_{rem} = \frac{k_1}{\sqrt{\ell_o}} + k_2 \qquad (5)$$

with k_1 and k_2 constants, comparable in magnetude. As we are considering variations of ℓ_o (from the AGS value) of a factor of 1/3 at most, and as the stray field components, which add linearly to the remanent field effects, comprise, for the existing machines, perhaps one-half of the field errors, we shall probably make errors of 20% or so in δB if we neglect the ℓ_o dependence for the present arguments.

It is now argued that one of the basic economic factors in the design of a high-energy accelerator is the size of the guide-field aperture, which in turn

---

** Note, however, that a small δh may be more easily achieved with small aperture magnets.

depends basically on the excursions expected of the closed orbit. One of the primary contributions to these excursions will come from the field errors and from errors in surveying-in the magnets. We propose that one boundary between "high" and "low" injection fields should be taken as that field for which the orbit excursions due to the field variations is equal to those which arise from magnet positioning errors.

If we let ε represent the expected r.m.s. positioning errors, then our boundary field B_1 is defined by

$$B_1 = \frac{\ell_o \, \delta B}{\varepsilon} \tag{6}$$

This field varies in proportion to the scale parameter ℓ_o. Using 0.10 gauss for δB and 0.013 cm (0.005") for ε, we have

$$B_1 = (11.5 \text{ gauss/cm}) \, \ell o \tag{7}$$

The graph of Fig. 1 has as coordinates the scale parameter ℓ_o and the injection field B_{inj}. (For pole profiles scaled according to the method of SL-10, the radial aperture a is very nearly $\ell_o/2$.) The relation of Eq. (6) is the line marked "B_1". Fields below B_1 are "low" with respect to our criterion based on total field errors. The line (marked 0.01") indicates the function B_1 for magnet position tolerance twice as large.

III Gradient Errors

Small-aperture, high-energy machines will have high ν-values and will be, therefore, particularly sensitive to errors in the guide field gradient. Gradient errors are likely to be particularly serious for guide fields which must have a large "dynamic range", i.e., those which operate from low injection energies.

For the same reasons given in (1) above, we consider here only the effects due to remanent and stray fields. We neglect for the moment the fluctuations from magnet to magnet and assume that in each magnet the vertical field near the equilibrium orbit can be written

$$B = B_o(t)(1 + n_o x) + B'(1 + n'x). \tag{8}$$

The first term represents the ideal time-dependent component of the field which arises from the exciting current. The second term is the time-independent contribution from the remanent and stray field effects. We believe that these two terms can be taken as approximately independent.

The field of Eq. (8) has the effective n-value given by

$$n = \frac{B_o n_o + B' n'}{B_o + B'} \tag{9}$$

Considering n_o to be the design-center, high-field value, then the n-error at any injection field $B = (B_o + B')$ is

$$\Delta n = n' - n_o = \frac{B'(n' - n_o)}{B} \tag{10}$$

The principle effect of the gradient error is to change the value of ν (the number of betatron oscillations per revolution). Since ν varies nearly as n,

$$\frac{\Delta \nu}{\nu_o} = \frac{\Delta n}{n_o} \tag{11}$$

where ν_o and n_o refer to the design values.

Using (10),

$$\frac{\Delta \nu}{\nu_o} = \frac{B'}{B}\left(\frac{n'}{n_o} - 1\right) \tag{12}$$

The scaling rules adopted in SL-10 give a relation between ν_o, ℓ_o, and E, the maximum proton energy,

$$\nu_o = K(E/\ell_o)^{1/2} \quad , \tag{13}$$

with $K = 8.75\ (24\ \text{cm}/30\ \text{Gev})^{1/2}$.

Eqs. (12) and (13) can be used to obtain (for any particular final energy) a relation between ℓ_o and B_2 the injection field for which the betatron frequency differs by $\Delta \nu$ from its high-field value

$$B_2 = K\left(\frac{E}{\ell_o}\right)^{1/2} \frac{B'}{\Delta \nu}\left(\frac{n'}{n_o} - 1\right) \tag{14}$$

The dependence of B_2 on ℓ_o is in the opposite sense of that of B_1, favoring high injection fields for small scale magnets.

The available CERN and BNL data give for the mean value of the remanent field, approximately 15 gauss. These data also give for the remanent field values for $\left(\frac{n'}{n_o} - 1\right)$ of about 0.10. It may be expected that the stray fields -- which have given rise to the unexpected coupling of radial and vertical oscillations observed at both CERN and BNL -- would also make a contribution to n'. No quantitative data are available on this point. For our purposes, we adopt the nominal value

$$B' = 15 \text{ gauss}$$

$$\left(\frac{n'}{n_o} - 1\right) = 0.10 \qquad (15)$$

A "significant" $\Delta\nu$ would be one which moved the operating point close to a nearby resonance. We suggest taking $\Delta\nu = 0.2$ (approximately three quarters of the way from the operating point to the nearest major resonance) as determining the dividing line between "high" and "low" injection fields -- relevant to the consideration of gradient errors.

The relation between B_2 and ℓ_o obtained from Eq. (14) using the numerical values of (15), and setting $\Delta\nu = 0.2$, are shown for a 300 Gev machine by the curve B_2 in Fig. 1. Corresponding relations for other energies are indicated by the dashed curves.

Fluctuations of the field gradient along the orbit will give rise to stop bands at the resonances which decrease the region of the stability diagram in which bounded orbits are obtained. Preliminary estimates of the stop-band widths expected (using Eq. 4.56 of Courant and Snyder[1]) are likely to be less significant than the effects considered above.

---

1) E. D. Courant and H. S. Snyder, Annals of Physics 1, 1 (1958).

IV Discussion

The functions B_1 and B_2 derived above and plotted in Fig. 1, each serve to define a boundary between "high" and "low" injection fields. For injection fields and aperture scales which correspond to points above these curves field errors at injection will have little influence on the operation. At injection fields below these curves -- "low" injection energies -- allowances must be made for the orbit distortions to be expected at injection either by increasing the aperture dimensions, or by providing for field corrections.

It is not intended to suggest that "high" or "low" injection fields are "good" or "bad", or, in particular, that "high" injection fields necessarily offer any advantages. The terms are used here in the following senses:

> "High" injection fields are those for which field errors at injection do not add to the required aperture. The cost of the injector must be justified by factors other than the scale of magnet (e.g., intensity).
>
> "Low" injection fields are those for which the aperture requirements at injection are strongly influenced by the choice of the injection energy.
>
> The relation of the cost of the injector to the cost of the guide magnet must be carefully considered.

Careful economic arguments may lead to an optimum design for any particular machine with an injection field which is either "high" or "low".

The injection field and aperture for the orbit parameters of SL-10 correspond to the point so marked in the figure. The injection occurs at a "high" field and the neglect, in the early considerations, of perturbing effects at injection was justified. For comparison, the A.G.S. parameters (also indicated on the figure) place it at an intermediate field -- below B_1, but above the B_2 for 30 Gev. It should be recalled, however, that the original design anticipated magnet placement errors of 0.02 inch. In this case, the design would fall in the "high" field region.

It is apparent from Fig. 1 that the injection energy of SL-10 is probably higher than would be necessary merely to avoid difficulties from field irregularities at injection. The use of a higher current injector (high pulse rate, linac, etc.) would probably introduce economic arguments in favor of lower injection energies. In this case, it would appear that an injection field near 300 gauss with an $_{o}$ of 12 to 15 cm (a radial aperture of 6.0 to 7.5 cm) should be seriously considered.

It also appears from the considerations of this note that if a synchrotron larger than 300 Gev is contemplated, it will require higher injection energies, a larger aperture, more careful field corrections, or some combination of all three.

CAPTION TO FIGURE 1.

Figure 1. Relation between injection parameters. B_{inj} is the injection field, ℓo is the scale parameter of the magnet aperture ($\ell o = \rho/n$). For apertures scaled from the A.G.S. design (according to the method of SL-10) ℓo is about twice the horizontal aperture. B_1 is the injection field for which the random errors in the remanent field produce excursions of the equilibrium orbit equal to those produced by random magnet placement errors of r.m.s. amplitude $\varepsilon = 0.01''$. B_2 is the injection field for which the gradient errors at injection produce a shift of ν of 0.15.

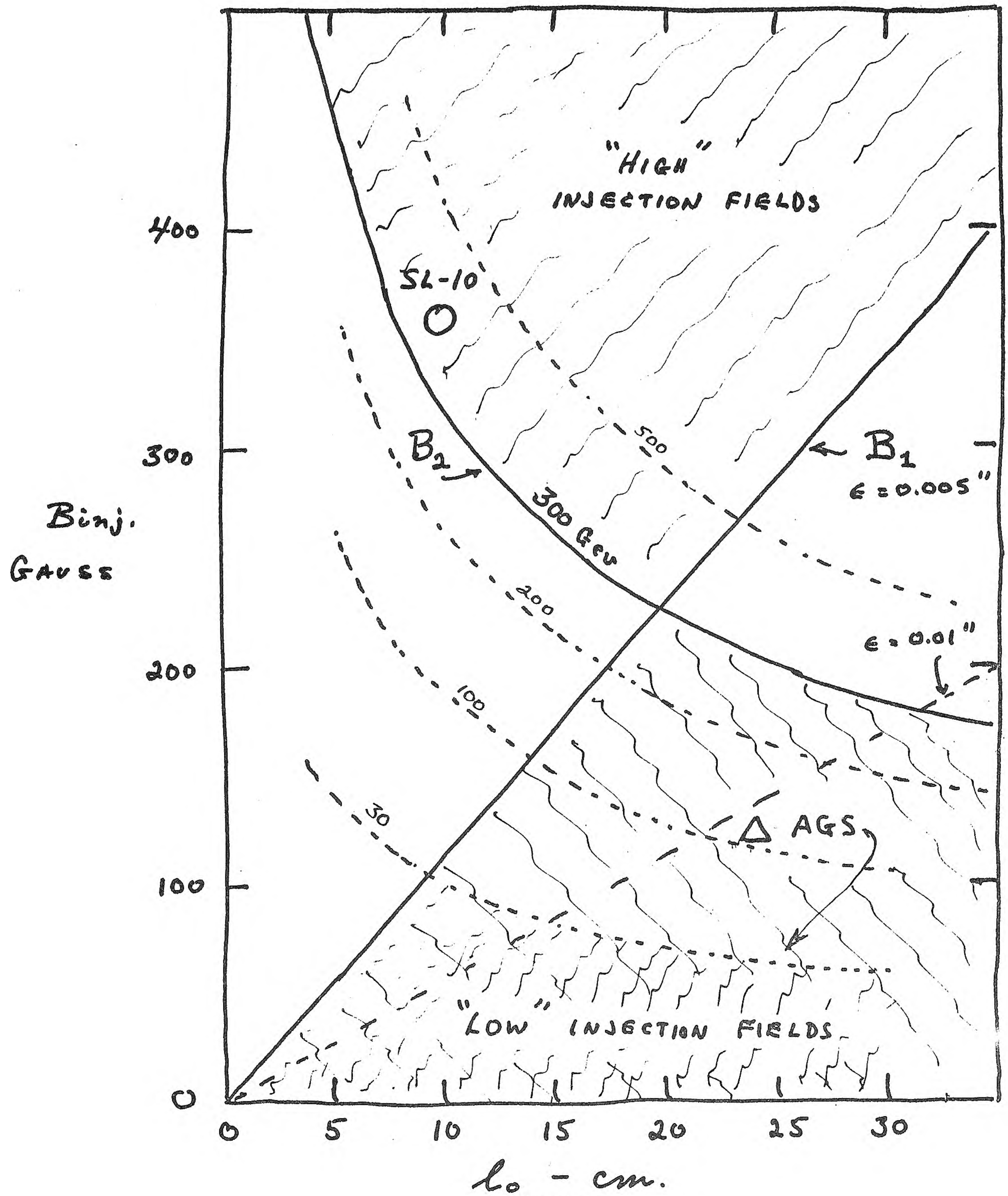
"HIGH"
INJECTION FIELDS
SL-10
B_2
300 GeV
500
200
100
30
B_1
ε = 0.005"
ε = 0.01"
AGS
"LOW" INJECTION FIELDS
400
300
200
100
0
Binj.
GAUSS
0
5
10
15
20
25
30
l_o - cm.

FIGURE 1